Full-Wave Analysis of Guided Wave Structures Using a Novel 2-D FDTD

S. Xiao, R. Vahldieck, and H. Jin

Abstract— **A two-dimensional Yee's mesh with reduced grid size is proposed for the full-wave analysis of arbitrarily shaped guided wave structures. By introducing a phase shift $\beta\Delta h$ along the z-direction (propagation direction), it is now possible to calculate the propagation constant of hybrid modes by using only a two-dimensional mesh. This step not only allows the frequency selective application of the finite difference time-domain (FDTD) method, as desired in many design problems, but it also reduces the memory space and CPU time of the full-wave FDTD significantly. Furthermore, by introducing a phase shift, the size of the space grid in propagation direction is reduced to half its normal size.**

I. Introduction

When Yee [1] introduced the FDTD, he discretized Maxwell's equations directly by the central difference formula in time and space. Since then, the FDTD has been further developed and is now well established as a versatile technique to solve electromagnetic field problems. The method is in particular attractive for transmission line problems with complicated circuit contours. Application examples have been reported in i.e., [2]–[9]. Although the method has many attractive features for time-domain problems, one commonly known disadvantage of the FDTD if utilized in frequency selective analysis problems is, that it requires large amounts of memory space and CPU time, in particular for the full-wave analysis of hybrid modes in quasiplanar circuits or in general in inhomogeneous waveguide structures.

The large memory space and CPU time requirements are mainly due to the fact that the full-wave analysis requires a three-dimensional mesh and that processing a time-domain impulse involves from the start much more frequency information than what is actually needed for the circuit analysis. Only after the impulse has reached stability in the three-dimensional mesh, a Fourier transform selects the information of interest.

To improve the computation time and memory space requirements of the FDTD, it is crucial to reduce its mesh size and preselect the frequency range of interest to avoid processing of unecessary information. However, up to now a reduced mesh size (two-dimensional) could only be used to calculate the TM or TE mode case separately [1], [7], [8]. Although several other slightly different approaches for the FDTD have been reported, all of them require a three-dimensional mesh to determine hybrid modes. For example, one of those methods uses a Gaussian pulse as excitation for a single shielded microstrip line. Typically 160 space meshes are required in propagation direction and about 5 to 7 time steps for any one mesh to satisfy the stability condition [5], [9]. Another approach is to resonate a section of the guided structure by placing two short-circuited planes along the z-axis a distance L apart. The length L corresponds to half a guided wavelength of the mode of interest. The resonance frequency of the cavity corresponds to the frequency at which this particular propagation constant is valid. The relationship between the propagation constant and L is then $\beta = 2\pi/L$. By changing L also β changes. Repeating the calculation for the resonance frequency of the resonator for each β, the dispersion characteristic of the guided structure can be obtained [3]. Because also this method involves a three-dimensional mesh, there are easily thousands of iteration steps involved.

To alleviate these problems, this letter introduces a novel approach for the FDTD that uses only a two-dimensional mesh consisting of a three-dimensional space grid for the analysis of hybrid modes. This two-dimensional mesh could also be regarded as one slice out of a three-dimensional mesh, with the third dimension, the propagation direction, being replaced by introducing a phase shift $\beta\Delta h$. This step even allows to reduce the size of the space grid to only half of its normal size. At a first glance, the introduction of a phase shift in the time-domain algorithm seems to be an odd approach. However, by choosing the propagation constant first and then exciting the system with a time-domain impulse provides correct results (after a Fourier transform) at the frequency at which this propagation constant is valid. This frequency corresponds to the first peak in the Fourier spectrum. Higher order modes correspond to the other peaks in the spectrum. This step must then be repeated for different propagation constants to obtain the dispersion curve for one particular mode. Since this approach requires only a two-dimensional mesh with a half-size space grid and since the propagation constant is given as an input parameter, the convergence rate is much faster than in the conventional approach and the memory space is reduced significantly.

II. The Two-Dimesional Mesh

The new approach follows the two-step leapfrog FDTD procedure initially developed for a full-sized three-dimensional grid. When the field components are normalized by the free space impedance $Z_0 = \sqrt{\mu_0/\varepsilon_0}$:

$$E^n_{x,y,z}(i,j,k) = e^n_{x,y,z}(i,j,k)\sqrt{Z_0}$$
$$H^{n+0.5}_{x,y,z}(i,j,k) = h^{n+0.5}_{x,y,z}(i,j,k)/\sqrt{Z_0}, \qquad (1)$$

Manuscript received October 23, 1991.
The authors are with the Laboratory for Lightwave Electronics, Microwave and Communications (LLiMiC), Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC, V8W 3P6 Canada.
IEEE Log Number 9200142.

1051–8207/92$03.00

one obtains (2) (see bottom of page) where $s = c\Delta t/\Delta h$, and Δt and Δh are, respectively, the time and space step. Here the stability condition requires

$$s \leq 1/\sqrt{3}. \tag{3}$$

When the modes have been established a period of time after the excitation, only a phase shift $\beta\Delta h$ is involved at any adjacent nodes for any specific propagation constant β. This modal knowledge is now used to simplify the scheme. It is easy to see that any incident or reflected field impulse for any propagation constant β satisfies [11]

$$\begin{aligned} e_p^n(i,j,k\pm 1) &= e_p^n(i,j,k)\exp\{\mp j\beta\Delta h\} \\ h_p^n(i,j,k\pm 1) &= h_p^n(i,j,k)\exp\{\mp j\beta\Delta h\}, \qquad p = x, y, \end{aligned} \tag{4}$$

then (2) can be rewritten as (5) (see bottom of page).From these equations it is obvious that now only a two-dimensional mesh is involved. Since this process closes the z-direction (Fig. 1), only a reduced space grid of half size remains. Moreover, no absorbing wall or shorted-shielding is needed along the propagation direction. The condition of stability is now found to be

$$s \leq \sqrt{2}. \tag{6}$$

The grid size in z-direction is arbitrary as long as the transmission line is homogeneous in this direction.

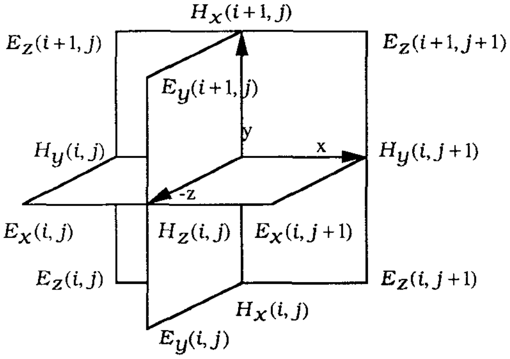

Fig. 1. Novel 2–D FDTD mesh.

III. Numerical Result

Calculations and comparisons have been made to verify the new approach. First, the air- and dielectric-filled rectangular waveguide is analyzed because of the availability of analytical results. The dominant mode is chosen as excitation. For the Fourier transform, the Blackman window function has been selected because of its very smooth shape. The CPU-time for this kind of problem was a few seconds on a Sun Sparc station. More complex structures can be handled easily. Fig. 2 shows the dispersion curves for two coupled dielectric guides and a comparison with the TLM [11] and mode matching method (MMM) [12]. While all three methods are in close agreement, the CPU-time for the MMM to calculate one frequency point is in the order of hours (on a Sun Sparc station) and that of the condensed node TLM, as modified in [11], is about 1 to 2 minutes. The new FDTD approach consumed only around 10 seconds of the CPU-time because only a half mesh is involved here.

IV. Conclusion

A 2-D FDTD mesh for the full-wave analysis of inhomogeneous transmission lines has been introduced. Using only a half-size space grid the memory space and CPU-time of the FDTD has been reduced significantly. Introducing a phase

$$\begin{aligned} h_x^{n+0.5}(i,j,k) &= h_x^{n-0.5}(i,j,k) - s\{e_z^n(i,j,k) - e_z^n(i,j+1,k) - e_y^n(i,j,k+1) + e_y^n(i,j,k)\} \\ h_y^{n+0.5}(i,j,k) &= h_y^{n-0.5}(i,j,k) - s\{e_x^n(i,j,k+1) - e_x^n(i,j,k) - e_z^n(i+1,j,k) + e_z^n(i,j,k)\} \\ h_z^{n+0.5}(i,j,k) &= h_z^{n-0.5}(i,j,k) - s\{e_y^n(i+1,j,k) - e_y^n(i,j,k) - e_x^n(i,j+1,k) + e_x^n(i,j,k)\} \\ e_x^{n+1}(i,j,k) &= e_x^n(i,j,k) + s\{h_z^{n+0.5}(i,j,k) - h_z^{n+0.5}(i,j-1,k) - h_y^{n+0.5}(i,j,k) + h_y^{n+0.5}(i,j,k-1)\} \\ e_y^{n+1}(i,j,k) &= e_y^n(i,j,k) + s\{h_x^{n+0.5}(i,j,k) - h_x^{n+0.5}(i,j,k-1) - h_z^{n+0.5}(i,j,k) + h_z^{n+0.5}(i-1,j,k)\} \\ e_z^{n+1}(i,j,k) &= e_z^n(i,j,k) + s\{h_y^{n+0.5}(i,j,k) - h_y^{n+0.5}(i-1,j,k) - h_x^{n+0.5}(i,j,k) + h_x^{n+0.5}(i,j-1,k)\}, \end{aligned} \tag{2}$$

$$\begin{aligned} h_x^{n+0.5}(i,j) &= h_x^{n-0.5}(i,j) - s\{e_z^n(i,j+1) - e_z^n(i,j) + 2\sin(\beta\Delta h/2)e^{j(\pi-\beta\Delta h)/2}e_y^n(i,j)\} \\ h_y^{n+0.5}(i,j) &= h_y^{n-0.5}(i,j) - s\{e_z^n(i,j) - e_z^n(i+1,j) - 2\sin(\beta\Delta h/2)e^{j(\pi-\beta\Delta h)/2}e_x^n(i,j)\} \\ h_z^{n+0.5}(i,j) &= h_z^{n-0.5}(i,j) - s\{e_y^n(i+1,j) - e_y^n(i,j) - e_x^n(i,j+1) + e_x^n(i,j)\} \\ e_x^{n+1}(i,j) &= e_x^n(i,j) + s\{h_z^{n+0.5}(i,j) - h_z^{n+0.5}(i,j-1) - 2\sin(\beta\Delta h/2)e^{-j(\pi-\beta\Delta h)/2}h_y^{n+0.5}(i,j)\} \\ e_y^{n+1}(i,j) &= e_y^n(i,j) + s\{h_z^{n+0.5}(i-1,j) - h_z^{n+0.5}(i,j) + 2\sin(\beta\Delta h/2)e^{-j(\pi-\beta\Delta h)/2}h_x^{n+0.5}(i,j)\} \\ e_z^{n+1}(i,j) &= e_z^n(i,j) + s\{h_y^{n+0.5}(i,j) - h_y^{n+0.5}(i-1,j) - h_x^{n+0.5}(i,j) + h_x^{n+0.5}(i,j-1)\}. \end{aligned} \tag{5}$$

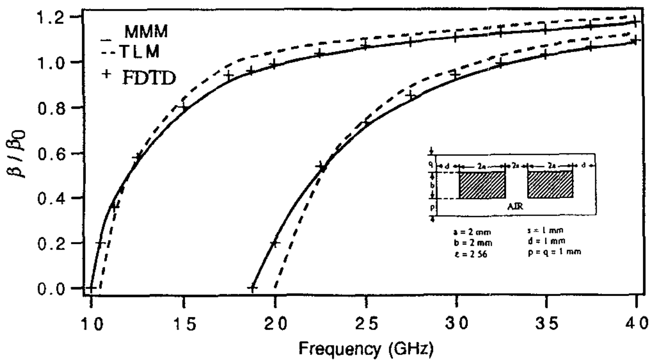

Fig. 2. Numerical comparison between 2-D FDTD, mode-matching method, and transmission-line-matrix method.

shift in axial direction and chosing the propagation constant as input parameter, allows a frequency selective application of the FDTD. This makes the FDTD a very efficient tool for practical CAD of various complicated microwave circuits.

REFERENCES

[1] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302–307, May 1966.

[2] X. Zhang, J. Fang, K. K. Mei, and Y. Liu, "Calculations of the dispersive characteristics of microstrips by the time-domain finite difference method," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 263–267, Feb. 1988.

[3] D. H. Choi and W. J. R. Hoefer, "The finite difference time-domain method and its application to eigenvalue problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1464–1470, 1986.

[4] M. Rittweger and I. Wolff, "Analysis of complex passive (M)MIC-components using the finite difference time-domain approach," *IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 1147–1150, 1990.

[5] X. Zhang and K. K. Mei, "Time-domain finite difference approach to the calculation of the frequency-dependent characteristics of microstrip discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1775–1787, 1988.

[6] W. K. Gwarek, "Microwave circuits designed by two-dimensional vector wave equation and their analysis by FDTD method," *21th European Microwave Conf. Dig.* (German), Sept. 1991, pp. 866–871.

[7] W. K. Gwarek, "Analysis of an arbitrarily-shaped planar circuit time-domain approach," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, no. 10, pp. 1067–1072, 1985.

[8] D. L. Paul, E. M. Daniel, C. J. Railton, "Fast finite difference time-domain method for the analysis of planar microstrip circuits," *21th European Microwave Conf. Dig.* (German), Sept. 1991, pp. 303–308.

[9] X. Zhang, "Time-domain finite difference analysis of the frequency-dependent characteristic of the microstrip discontinuities", Master thesis, Dept. of EECS, Univ. of California, Berkeley, Nov. 1987.

[10] A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing.* Prentice-Hall, 1975.

[11] H. Jin, R. Vahldieck, and S. Xiao, "An improved TLM full-wave analysis using a two-dimensional mesh," *IEEE MTT-S Int. Microwave Symp. Dig.*, Boston, MA, June 1991, pp. 675–677.

[12] A. K. Tiwari, B. Bhat and R. P. Singh, "Generalized coupled dielectric waveguide and its variants for millimeter-wave applications," *IEEE Trans. Microwave Tech.*, vol. MTT-34, pp. 869–875, Aug. 1986.